Introduction to eddy flux theory

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Outline

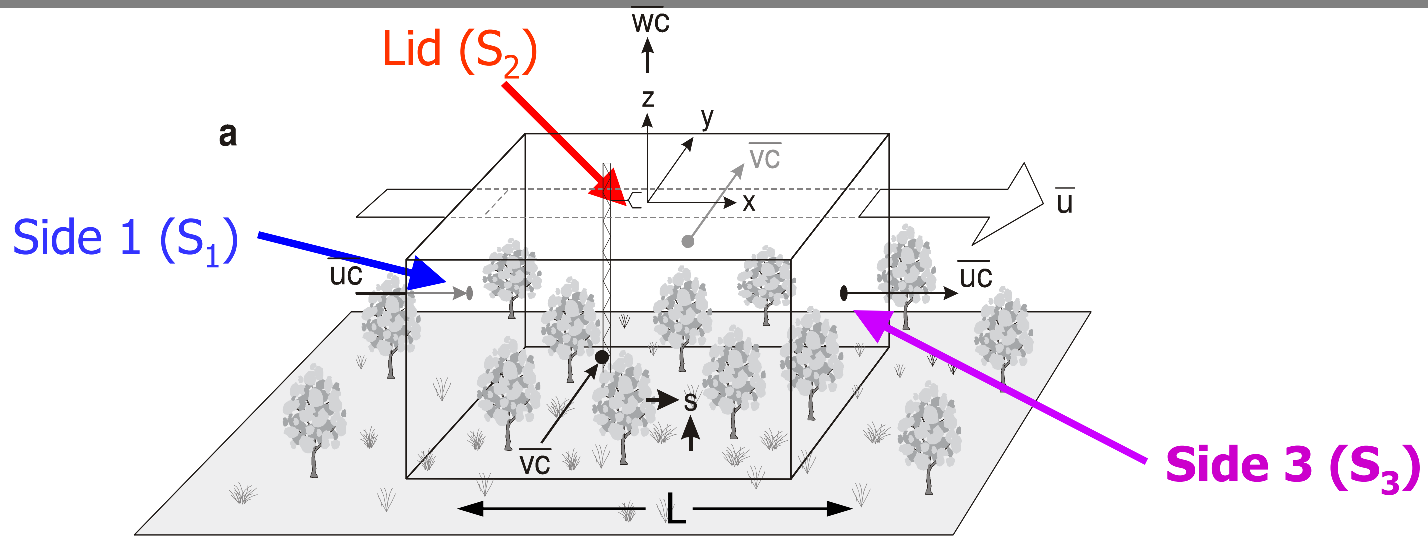
Mass balance of a control volume

Time vs spatial averaging concepts

Trace gas measurements

- Webb, Pearman & Leuning theory
- Open-path analysers
- Closed-path analysers

Conservation equation for Control Volume



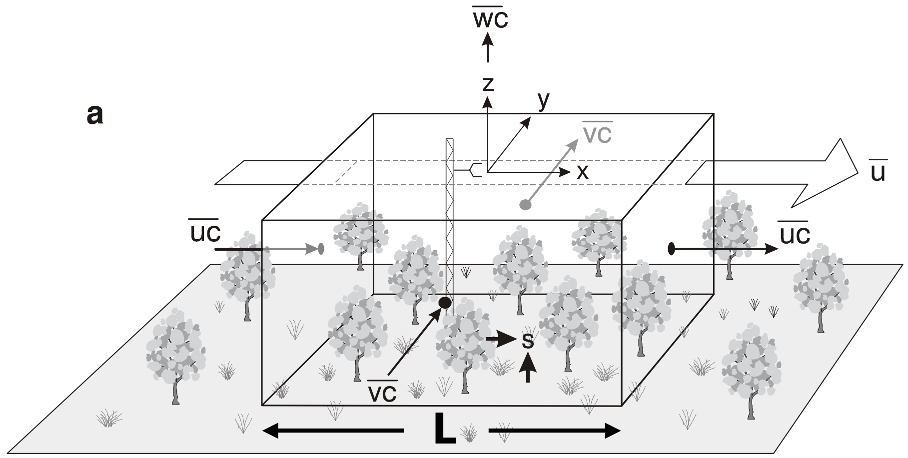
change in storage = Σ(Flux in) – Σ(Flux out)

$$\int_{dV} \overline{\frac{\partial \chi_c}{\partial t}}\, dV = L^2\, \overline{F_c}\, / \,\overline{c_d} + \int_{S_1} \overline{u\chi_c}\, dS_1 - \int_{S_2} \overline{w\chi_c}\, dS_2 - \int_{S_3} \overline{u\chi_c}\, dS_3$$

Surface flux

Note spatial integration

Mass balance on a control volume



$$\overline{F_c} = \overline{F_0} + \int_0^h \overline{S_c}\,dz = \frac{1}{L^2}\int_0^L\int_0^L\int_0^h \overline{c_d}\,\overline{\frac{\partial \chi_c}{\partial t}}\,dxdydz$$

$$+\frac{1}{L^2}\int_0^L\int_0^L\int_0^h \left[\overline{uc_d}\,\frac{\partial \overline{\chi_c}}{\partial x} + \overline{vc_d}\,\frac{\partial \overline{\chi_c}}{\partial y} + \overline{wc_d}\,\frac{\partial \overline{\chi_c}}{\partial z}\right] dxdydz$$

$$+\frac{1}{L^2}\int_0^L\int_0^L\int_0^h \left[\frac{\partial \overline{c_d}\,\overline{u'\chi_c'}}{\partial x} + \frac{\partial \overline{c_d}\,\overline{v'\chi_c'}}{\partial y} + \frac{\partial \overline{c_d}\,\overline{w'\chi_c'}}{\partial z}\right] dxdydz$$

Coordinate system

Have used rectangular Cartesian coordinates

Can rarely measure all components of mass balance.

To maximize information at tower choose site and coordinate system to ensure:

$$\int_{S_1} \overline{u\chi_c}\, dS_1 = \int_{S_3} \overline{u\chi_c}\, dS_3$$

Horizontal homogeneity – no advection

$$\overline{v} = \overline{w} = 0$$

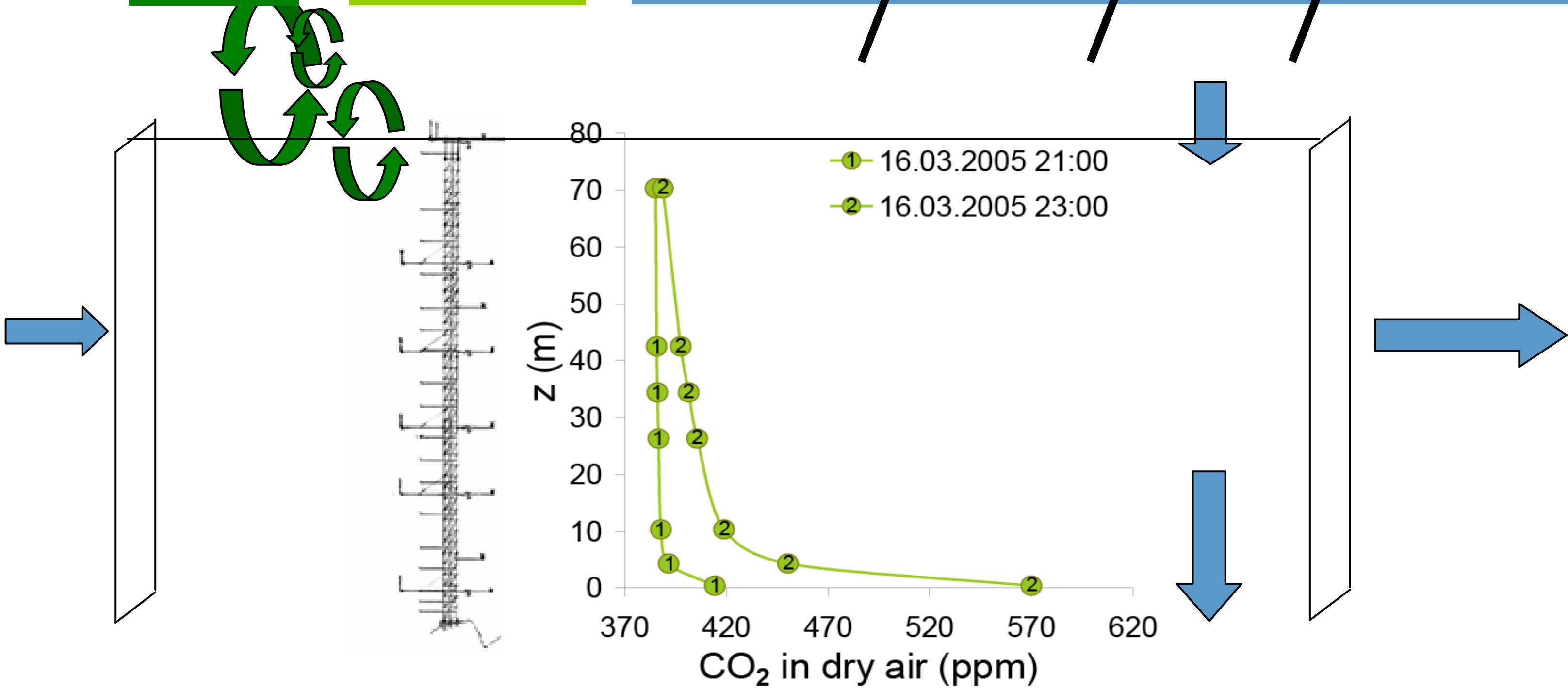
Coordinate rotation
(a topic in itself)

$$\overline{w\chi_c} = \overline{w}\,\overline{\chi_c} + \overline{w'\chi_c'} = \overline{w'\chi_c'}$$

Leaves only vertical eddy flux

Simplified mass balance equation

$$F_c = \overline{c_d}\,\overline{w'\chi_c'} + \int_0^h \overline{c_d}\frac{\partial \overline{\chi_c}}{\partial t}\,dz + \frac{1}{L^2}\int_0^L\int_0^L\int_0^h \left[\overline{uc_d}\frac{\partial \overline{\chi_c}}{\partial x} + \overline{vc_d}\frac{\partial \overline{\chi_c}}{\partial y} + \overline{wc_d}\frac{\partial \overline{\chi_c}}{\partial z} \right] dx\,dy\,dz$$



Horizontally homogeneous flow

Non steady-state change in storage + eddy flux

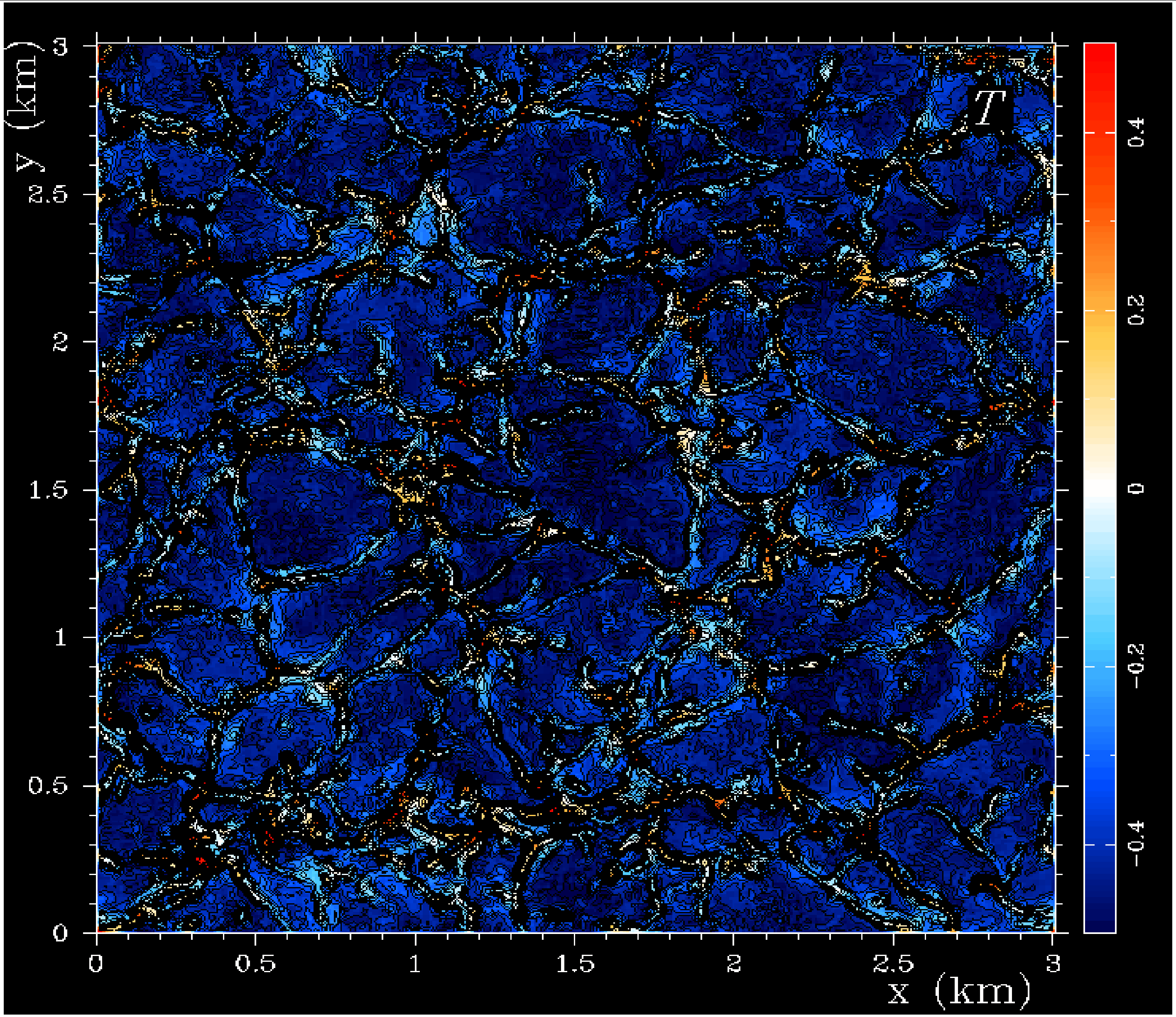
$$\overline{F_0} = \int_0^h \overline{c_d} \, \overline{\frac{\partial \chi_c}{\partial t}} dz + \overline{c_d} \, \overline{w' \chi_c'}$$

Steady-state eddy flux

$$\overline{F_0} = \overline{c_d} \, \overline{w' \chi_c'}$$

Time-averages at a point are assumed equal to spatial average – ergodic hypothesis

Assume time = space averaging

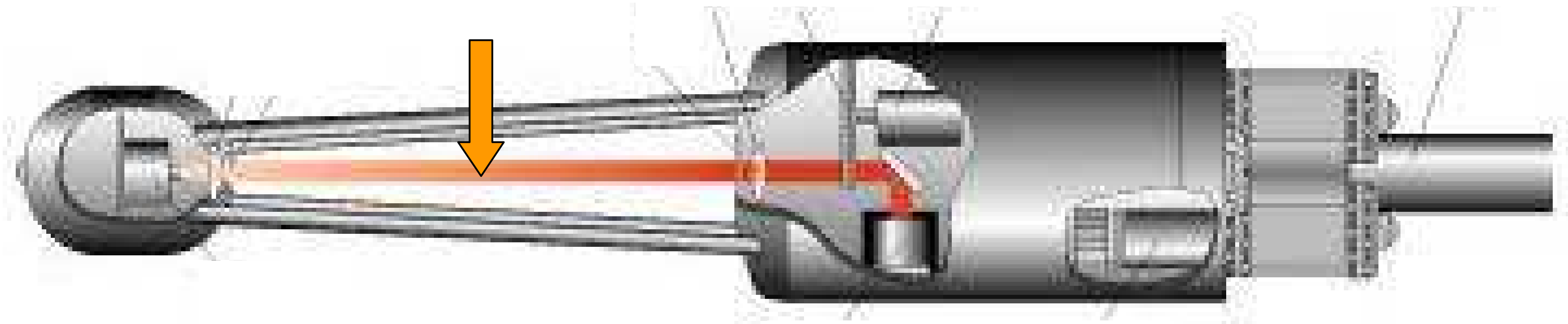


Tsutomu WATANABE

CO_2 and H_2O flux measurements

Licor 7500 Measures mol m^{-3} in optical path,

not required mixing ratios $\chi_v \ \chi_c$



But! Eddy fluxes have been expressed in terms of mixing ratio. What to do?

$$\overline{F_c} = \overline{c_d}\,\overline{w'\chi_c'}$$

Webb, Pearman & Leuning (1980) theory
Steady state, horiz. homogeneous flow

Can write trace gas flux using concentrations

$$\overline{F_c} = \overline{c_d}\,\overline{w'\chi_c'} \equiv \overline{w c_c} = \overline{w}\,\overline{c_c} + \overline{w'c_c'} \quad \textbf{but} \quad \overline{w} \neq 0$$

What is $\overline{w}$? WPL assumed no net flux of dry air

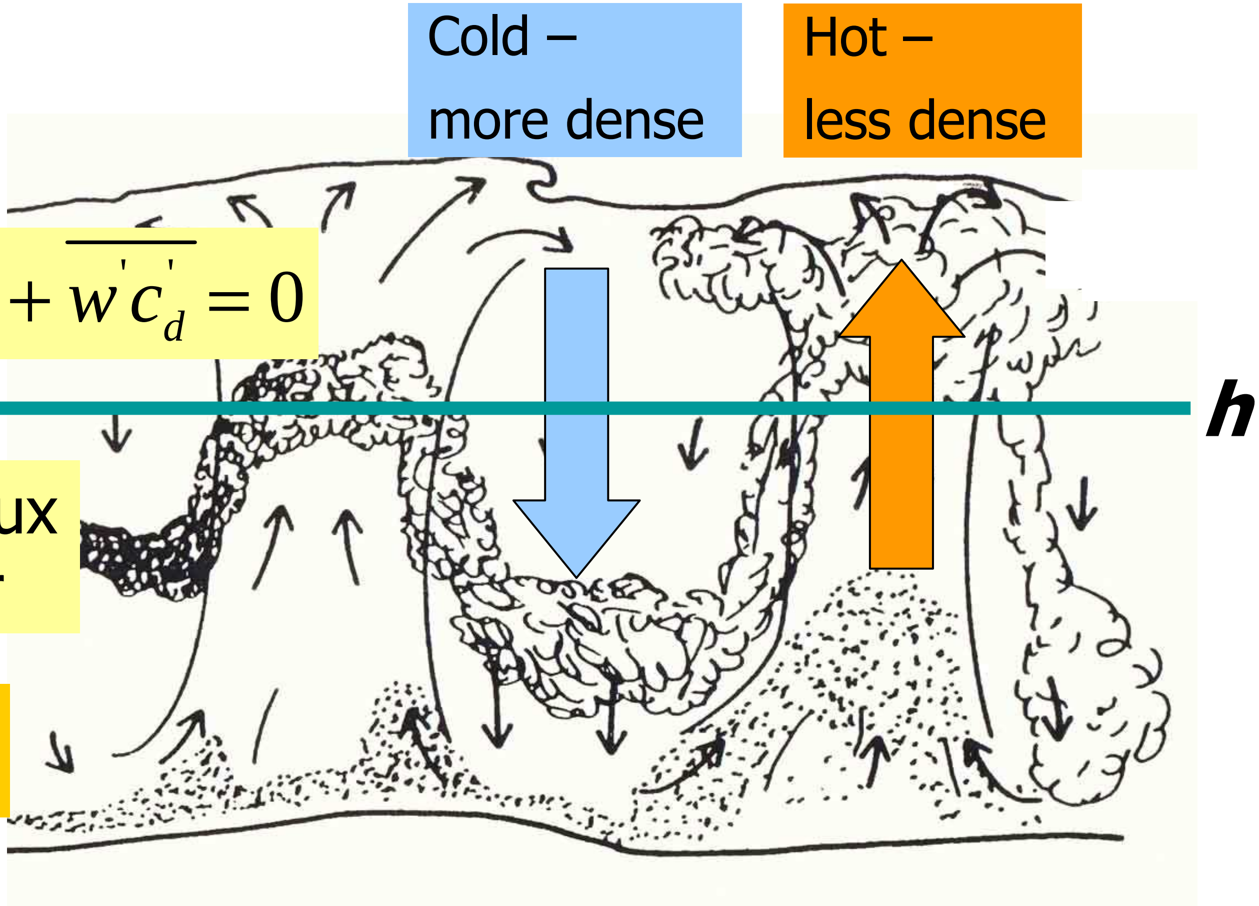
$$\overline{F_d} = 0 = \overline{w}\,\overline{c_d} + \overline{w'c_d'} \quad \Rightarrow \quad \overline{w} = -\overline{w'c_d'} \,/\, \overline{c_d}$$

Why is there a $\overline{w}$?

Consider ‘hot’ and ‘cold’ eddies over dry surface

$$\overline{F_d} = \overline{wc_d} = \overline{w}\,\overline{c_d} + \overline{w'c_d'} = 0$$

No net flux of dry air



$$\overline{w} = -\overline{w'c_d'}\,/\,\overline{c_d}$$

WPL theory

$$\overline{w} = -\overline{w'c_d'} / \overline{c_d}$$

Need expression for c_d'

WPL showed

$$\overline{w} = \frac{1}{\overline{c_d}}\left[\overline{w'c_v'} + \overline{c}\frac{\overline{w'T'}}{\overline{T}}\right]$$

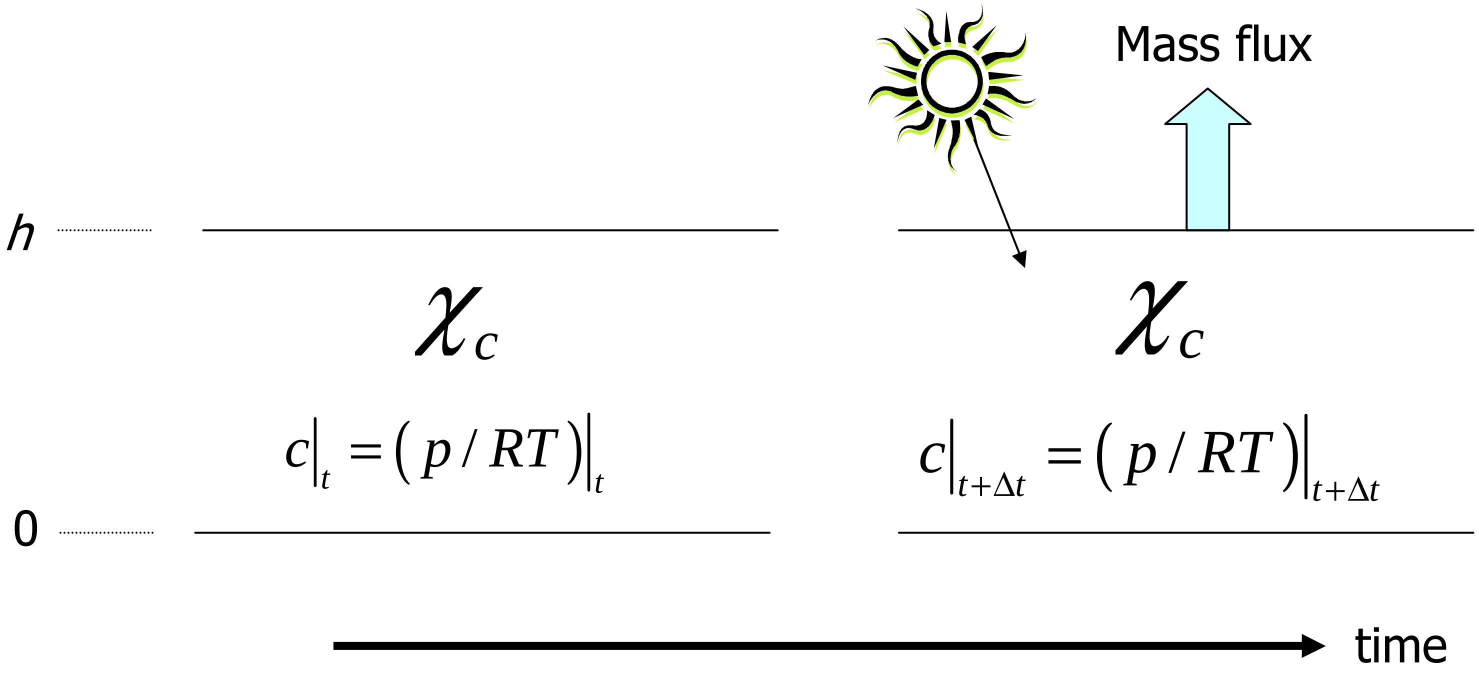
$< 3 \text{ mms}^{-1}$

Water vapor flux

Heat flux

Cannot measure $\overline{w}$ directly

What about non steady-state, horizontally homogeneous flow?



Change in concentration, but not mixing ratio

Eddy flux for trace gas

Leuning (2007) showed original WPL still correct

- No source/sink of dry air in the control volume

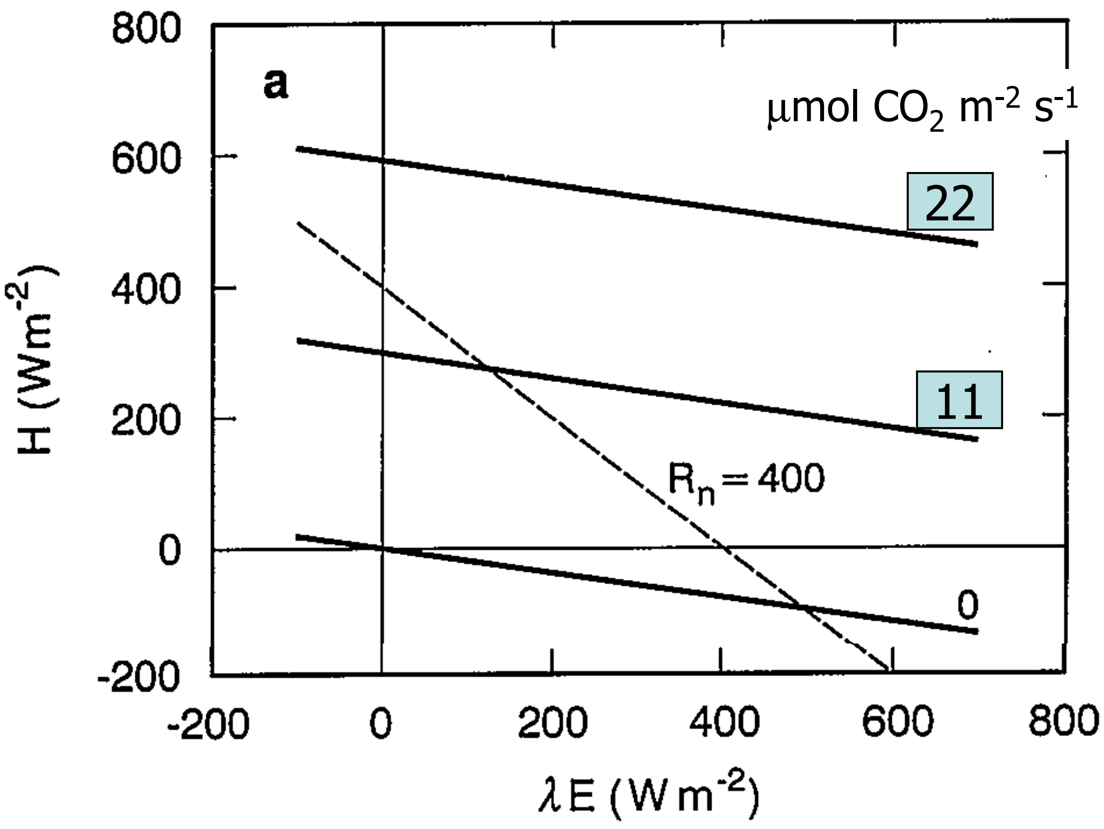
$$\overline{F_c} = \bar{c}_d\,\overline{w'\chi_c'} = \overline{w'c_c'} + \overline{\chi_c}\left[\overline{w'c_v'} + \bar{c}\,\frac{\overline{w'T'}}{\overline{T}}\right]$$

Raw CO_2 flux	Water vapor flux	Heat flux

Applies for horizontally homogeneous flow for both

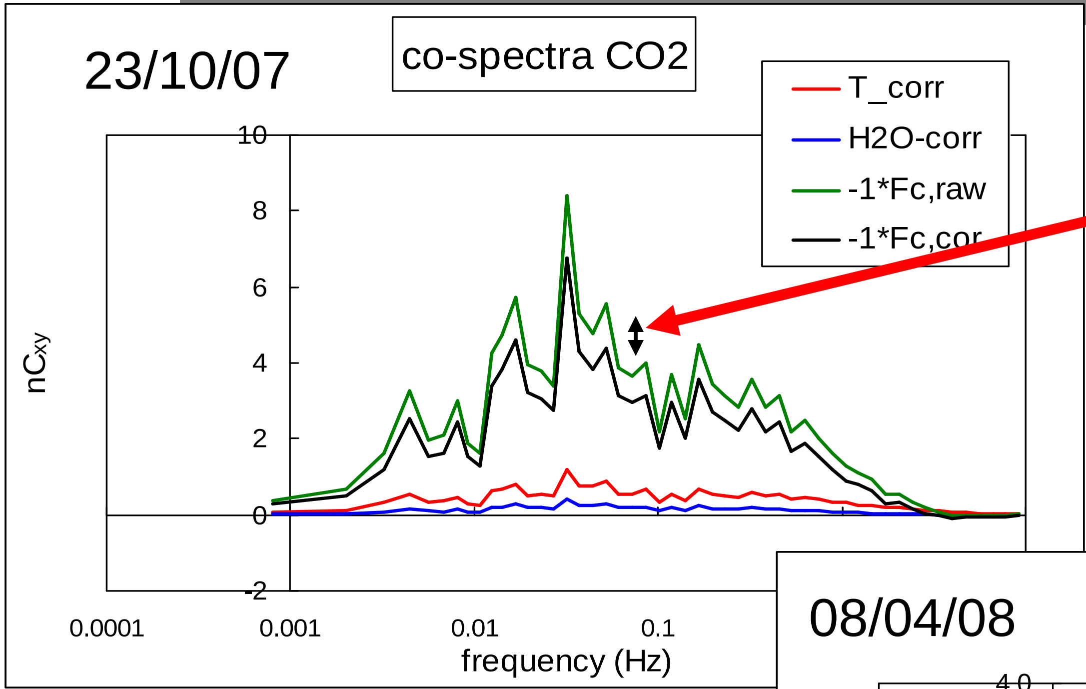
steady and non-steady conditions

Magnitude of WPL corrections – add to raw flux

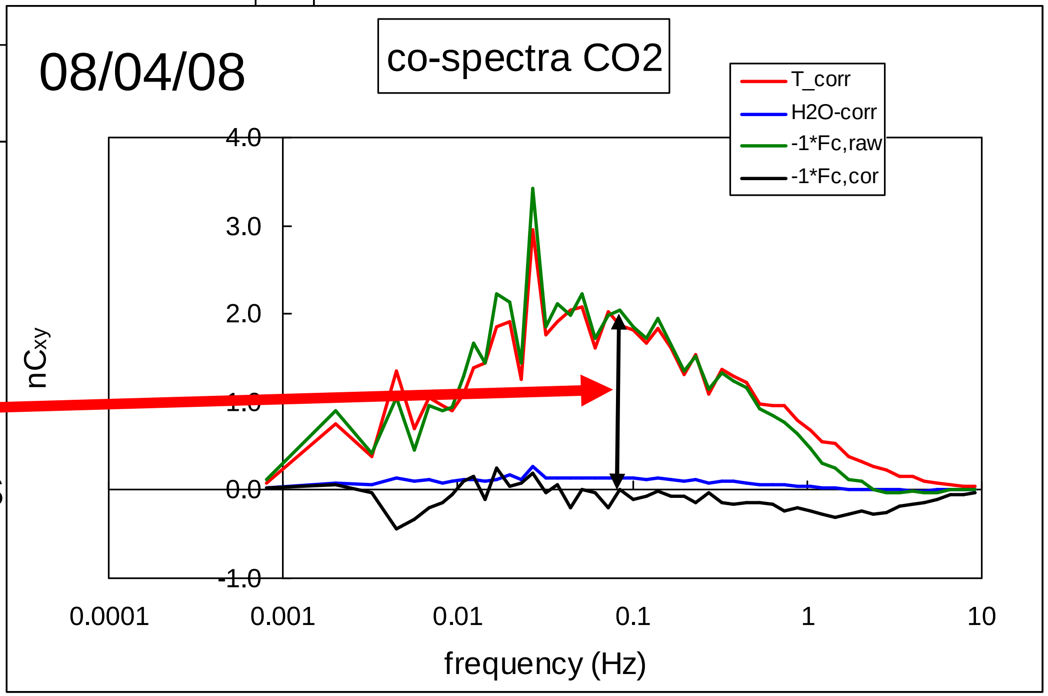


Leuning & Judd, 1996

WPL corrections to open path measurements



Small wT, wq
WPL corrections

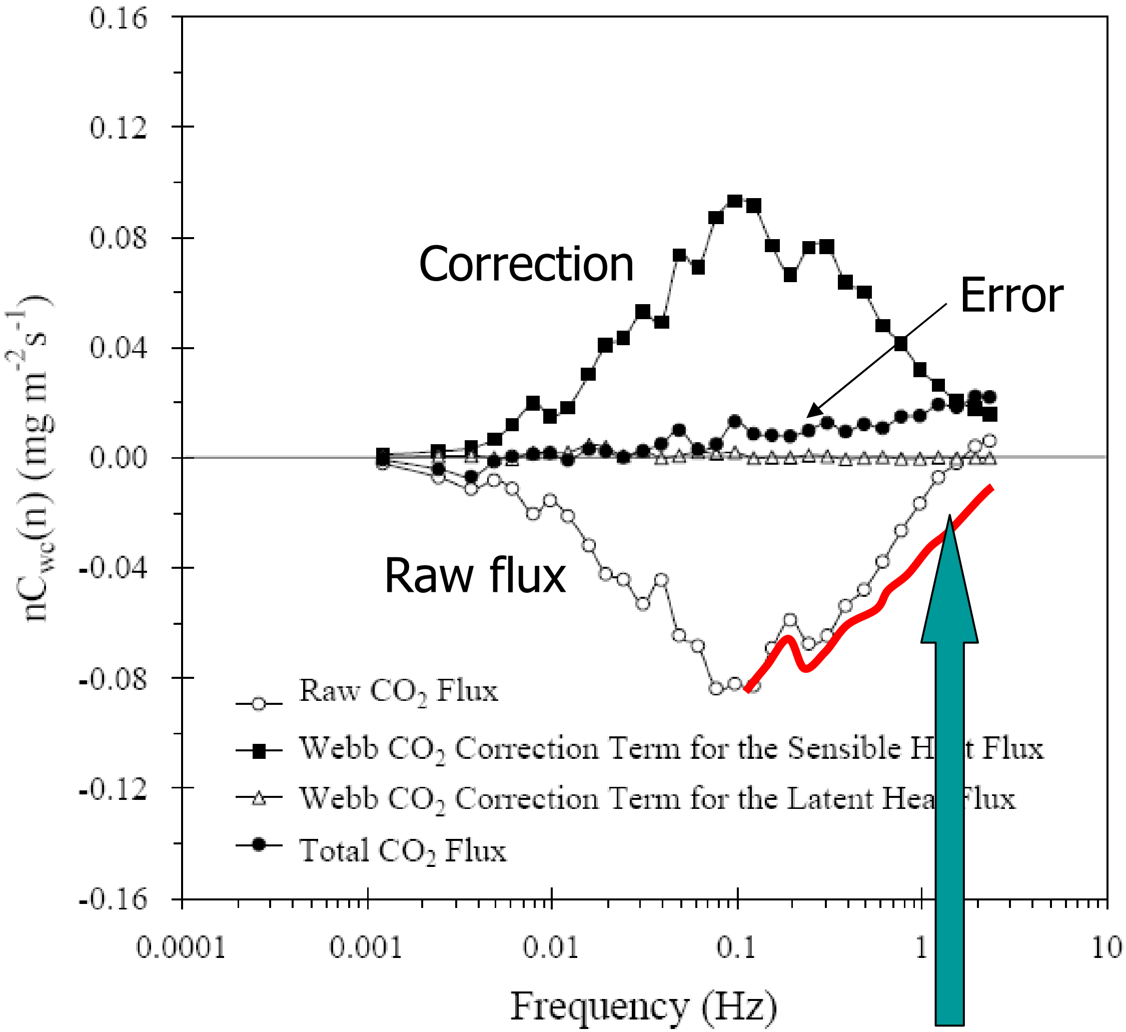


Big wT, wq

WPL corrections

Cospectra

Error due to differing frequency responses for cospectra of wT and wc_c



Need to correct for loss of covariance before WPL correction

Frequency response corrections

Define correction factor

$$C_F = \frac{\int_0^\infty C_{wc}(f)\,df}{\int_0^\infty G_{wc}(f)\,C_{wc}(f)\,df}$$

Numerator: ‘true’ cospectrum

Denominator: filtered cospectrum

G_{wc}: filter function – another topic

$C_F > 1$, typically

(Leuning and Moncrieff, 1990; Leuning & Judd 1996)

Open path measurements – calculation sequence

1) $\overline{H} = \overline{\rho} c_p \overline{w'T'}$

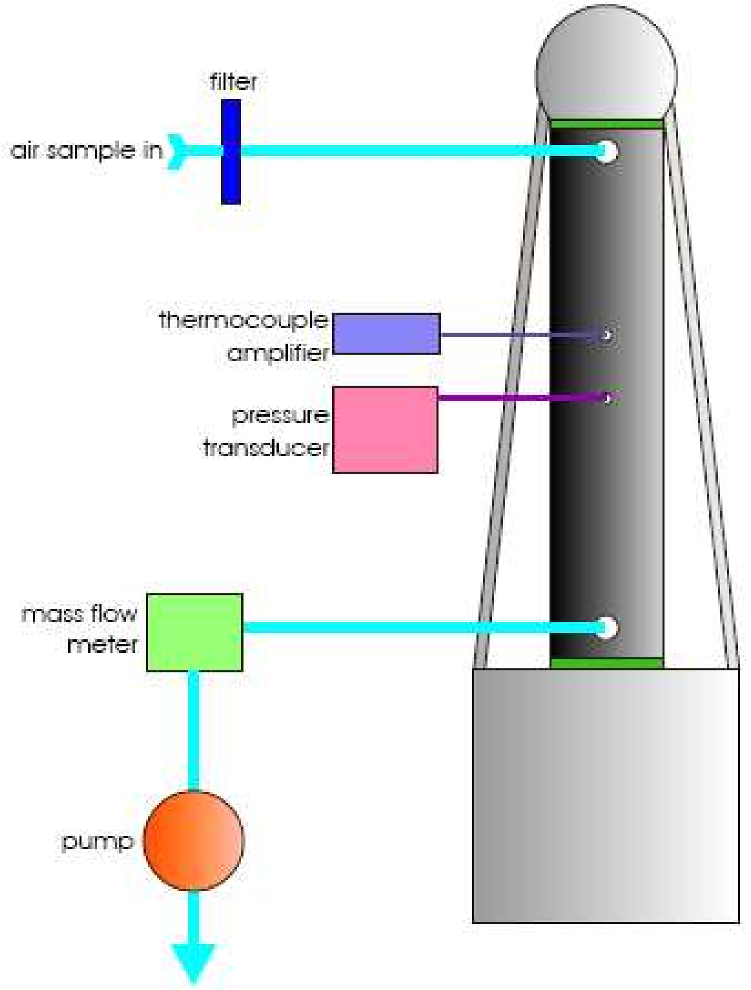
2) $\overline{E} = (1 + \overline{\chi}_v) \left[\overline{w'c_v'} + \frac{\overline{c}_v}{\overline{T}} \frac{\overline{H}}{\overline{\rho} c_p} \right]$

3) $\overline{F}_c = \overline{w'c_c'} + \overline{c}_c \left[\frac{\overline{E}}{\overline{c}} + \frac{\overline{H}}{\overline{\rho} c_p \overline{T}} \right]$

Assumes H, E & F_c have already been corrected for high & low frequency filtering

Closed-path analyser

Conversion of Li7500



Closed-path analyser

Measure c_c, c_v, T & P simultaneously in gas analyser and calculate mixing ratio at sampling rate used for eddy covariance

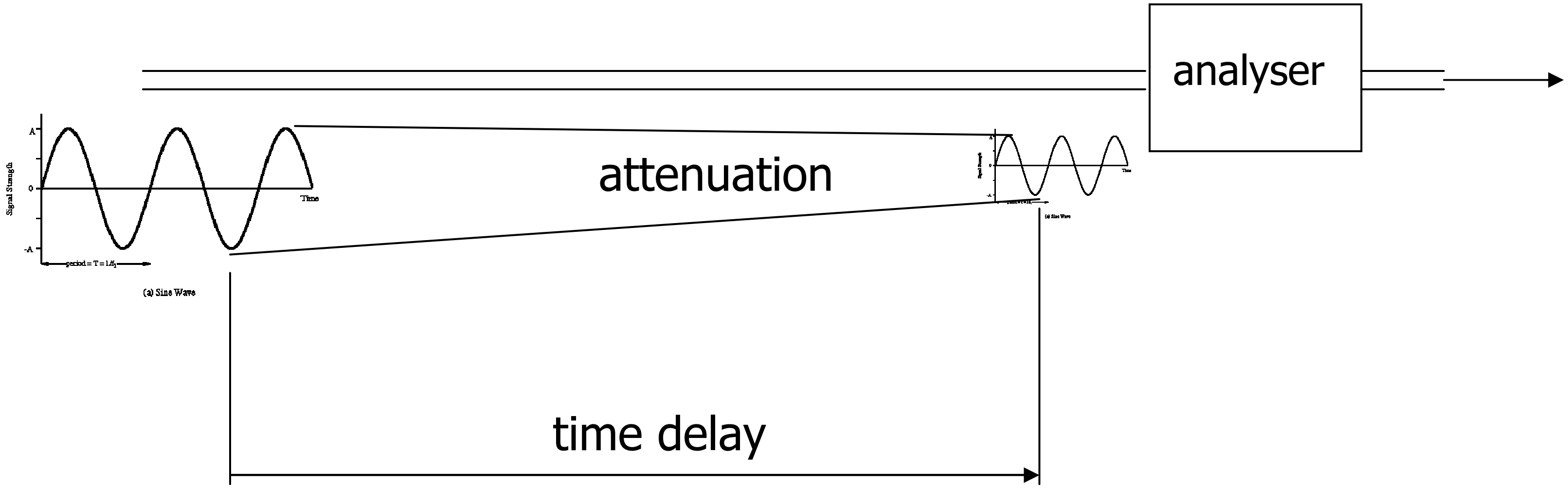
$$\chi_v = \frac{c_v}{P_i/(RT_i) - c_v}, \quad \chi_c = \frac{c_c}{P_i/(RT_i) - c_v}$$

Must also consider

- Time-lag
- Hi-frequency attenuation by air flow in tubing

CSIRO

Closed-path gas sampling



Tubing acts like a low-pass filter by mixing the air

Higher frequencies strongly attenuated – depends on:

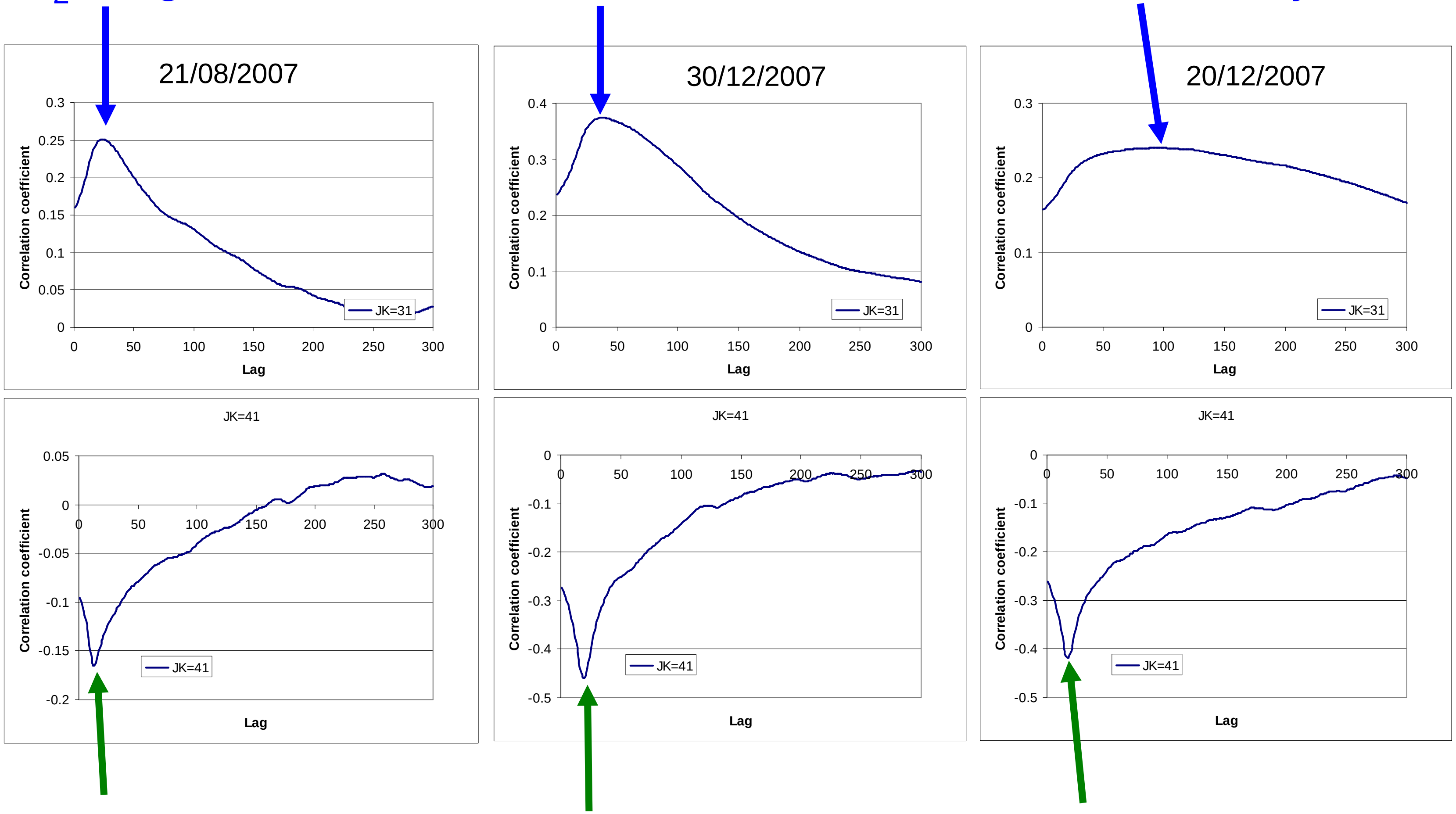
- Flow rate through tube
- Tube diameter, length and material

Lag at maximum correlation for closed path

H_2O lag @ max. correlation function of flow rate & rel. humidity

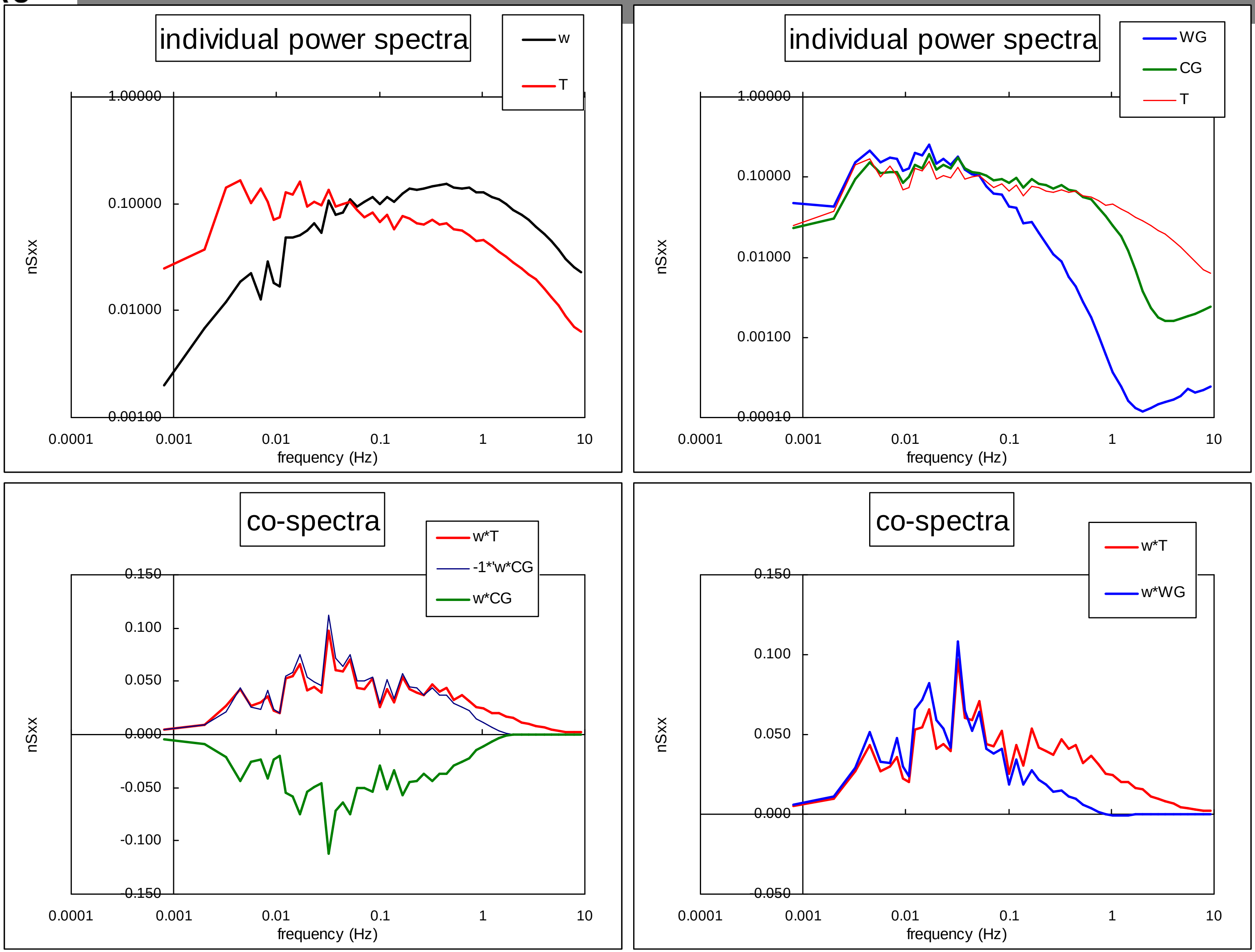
H_2O

CO_2



CO_2 lag @ max. correlation function of flow rate only

Closed path spectra and co-spectra



Summary

Mass balance of control volume -> surface flux

Choose site to avoid advection -> 1D transport

Time average a substitute for spatial average

Open-path analyser & WPL density corrections

- Want mixing ratio but measure density
- Density fluctuations due to temperature & humidity
- Corrections very large when H is large

Closed-path analysers

- Calculate mixing ratio
- Corrections needed for high-frequency attenuation
- Account for time lag