Using theory to make good measurements

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Design considerations for eddy flux measurements

- Measurement height
- Fetch/footprint - rule of thumb $z_m = x/100$
- Horizontal homogeneity of surface and topography
- High frequency filtering - instrumentation
- Low frequency filtering - averaging
- Application of WPL theory
- Open-path instruments
- Closed-path instruments
- Change in storage terms
Minimum measurements needed

Top of mast

- 3-D wind vector (20Hz)
- CO$_2$ and H$_2$O concentrations (20Hz)
- Net radiation
- Incoming solar radiation
- Reflected solar radiation
- RH and air temperature
- Rainfall
- Wind speed and direction
Minimum measurements needed (cont’d)

**Ground**
- Soil temperature
- Soil heat flux
- Soil moisture
- Rainfall
- Tree trunk temperature

**Profiles (multiple levels, 1 Hz)**
- Temperature
- CO$_2$
- H$_2$O vapour
- 2- or 3-D wind vectors
Tower schematic

- CO₂ Intake
- Cup Anemometer
- Precipitation Buckets
- Cup Anemometer
- Air Temperature Humidity Sensor
- CO₂ Intake
- Open Path CO₂/H₂O Analyzer
- Wind Vane
- CO₂ Intake
- Air Temperature Humidity Sensor
- CO₂ Intake
- Air Temperature Humidity Sensor
- CO₂ Intake
- Air Temperature Humidity Sensor
Horizontally homogeneous flow – no advection

\[ F_c = c_d \overline{w} \overline{\chi}_c + \int_0^h c_d \frac{\partial \overline{\chi}_c}{\partial t} dz + \frac{1}{L^2} \int_0^L \int_0^L \int_0^h \left[ \overline{u c_d} \frac{\partial \overline{\chi}_c}{\partial x} + \overline{v c_d} \frac{\partial \overline{\chi}_c}{\partial y} + \overline{w c_d} \frac{\partial \overline{\chi}_c}{\partial z} \right] dx dy dz \]
Internal boundary & equilibrium layers

- Fully Adjusted Layer
- Blending Height
- Internal Boundary
- Equilibrium Layer
- $h_B$

$Z_{02}$

$Z_{03}$

$X$
Height-to-fetch ratio

100:1 fetch rule of thumb

- Neutral conditions
- $> \text{ for stable conditions}$
- $< \text{ for unstable conditions}$

$z_m \leq X / 100$

Instrument placement

- Often a compromise between a representative footprint and avoiding advective effects
Typical eddy flux instrumentation

- Sonic anemometer
- Air intake for closed-path CO₂ & H₂O analyser
- Open-path CO₂ & H₂O analyser
High frequency attenuation

Line-averaging along instrument path
  - loss of variance

Spatial separation between instruments
  - loss of covariance
  - Samples eddies > ~2d
Variance spectrum - high-cut filter

\[ n = \frac{f z}{u} \]

\[ nS_{xx}/\sigma_x^2 \]

Ideal instrument

Non-ideal instrument

Missing spectral response
Covariance spectrum – high cut filter

![Graph showing measured cospectrum, true cospectrum, and missing cospectrum against frequency (Hz)]
Measurement height – a compromise

**System filter**

\[ f_c = \frac{\bar{u}}{d}, \]

\[ f_c \geq f_r \rightarrow z_m \geq 10d \]

**Atmospheric turbulence**

\[ f_r > 10\bar{u} / z_m, \]

\[ z_m \leq X / 100 \]

Remember equilibrium layer
Low frequency covariance

Average for long enough to
  - $u$ and $x$ axis are parallel to the ground
  - $z$ is normal to the ground
  - include all significant low-frequency contributions to the covariance

Averaging period increases with
  - measurement height
  - free convection (unstable boundary layers)
  - complex topography
Usual 30-min period may be too short to capture all the significant low frequency covariance. Convective conditions at Manaus tropical forest site ensure significant low frequency content in the covariance. This is lost if the averaging period is < ~4 hours.
Measurements on a single tower

\[ F_0 = c_d \int_0^h \frac{\partial \chi_c}{\partial t} \, dz + c_d \, \bar{w} \, \bar{\chi}_c \]

Vertical eddy flux $w' \, \bar{\chi}_c$

Change in storage

\[ \int_0^h \left[ c_d \frac{\partial \bar{\chi}_c}{\partial t} \, dt \right] \, dz \]
Sonic anemometer gives:

\[ u, v, w, u', v', w' \]

\[ H = \rho c_{pd} w' T'_v \]

Where sonic virtual temperature is

\[ T'_v = T(1 + 0.514q) \]

Still require

\[ \lambda E = \lambda c_d w' \chi'_v \]

\[ F_c = c_d w' \chi'_c \]
CO$_2$ and H$_2$O flux measurements

Licor 7500 Measures mol m$^{-3}$ in optical path, not required mixing ratios $\chi_v \chi_c$

But! Eddy fluxes have been expressed in terms of mixing ratio. What to do?

$$\overline{F_c} = c_d \overline{w'} \chi_c$$
Eddy flux for trace gas – WPL

\[
\overline{F_c} = c_d \overline{w' \chi_c} = \overline{w' c_c} + \chi_c \left[ \overline{w' c_v} + c \frac{\overline{w'T'}}{T} \right]
\]

Raw CO₂ flux  Water vapor flux  Heat flux

Applies for horizontally homogeneous flow for both steady and non-steady conditions
Error due to differing frequency responses for cospectra of \( wT \) and \( wc_c \)

Need to correct for loss of covariance before WPL correction
Open path measurements –
calculation sequence

1) \( \overline{H} = \rho c_p \overline{w'T} \)

2) \( \overline{E} = (1 + \overline{\chi_v}) \left[ \overline{w'c_v} + \frac{c_v}{T} \overline{H} \overline{\rho c_p} \right] \)

3) \( \overline{F_c} = \overline{w'c_c} + \overline{c_c} \left[ \overline{E} + \frac{\overline{H}}{\rho c_p T} \right] \)

Assumes \( H, E \) & \( F_c \) have already been corrected for high & low frequency filtering
Closed-path analyser

Conversion of Li7500
Closed-path analyser

Measure $c_c$, $c_v$, $T$ & $P$ simultaneously in gas analyser and calculate mixing ratio at sampling rate used for eddy covariance

\[
\chi_v = \frac{c_v}{P_i/(RT_i) - c_v}, \quad \chi_c = \frac{c_c}{P_i/(RT_i) - c_v}
\]

Must also consider

- Time-lag
- Hi-frequency attenuation by air flow in tubing
Tubing acts like a low-pass filter by mixing the air.
Higher frequencies strongly attenuated – depends on:

- Flow rate through tube
- Tube diameter, length and material
Closed path spectra and co-spectra

**individual power spectra**

![Graph 1](image1)

![Graph 2](image2)

**co-spectra**

![Graph 3](image3)

![Graph 4](image4)
Open vs closed path instruments

H₂O flux comparison
11-Aug-07 to 30 Jun-08

\[ y = 1.13x + 12.54 \]
\[ R^2 = 0.88 \]

CO₂ flux comparison
11-Aug-07 to 30 Jun-08

\[ y = 1.05x - 0.64 \]
\[ R^2 = 0.89 \]
Measurements on a single tower – change in storage term

Vertical eddy flux $w' \chi_c$

Change in storage

\[
\int_0^h \left[ \frac{\partial \chi_c}{\partial t} \right] \, dt \, dz
\]
Profiles of $T, c_c, c_v, u, v$

Biomass

$$\int_0^h \left[ \frac{\partial T}{\partial t} \right] dt \, dz$$

$\text{CO}_2 \& \text{H}_2\text{O}$
CO$_2$ & T profiles – change in storage term

\[ F_{\Delta storage} = \frac{C_d}{\Delta t} \left[ \int_{0}^{h} \chi_c \, dZ \right]_{t=\Delta t} - \left[ \int_{0}^{h} \chi_c \, dZ \right]_{t=0} \]
Energy balance

\[ H + \lambda E = R_S^{\downarrow} - R_S^{\uparrow} + R_L^{\downarrow} - R_L^{\uparrow} - G_0 - \Delta J_c \]

\[ H + \lambda E = R_n - G_0 - \Delta J_c \]
Soil heat flux

\[ \Delta J_s = \frac{m_s c_s}{\Delta t} \left[ T_s(z)dz\bigg|_{t+\Delta t} - T_s(z)dz\bigg|_t \right] \]
Magnitude of biomass heat storage term
Canopy Energy Balance - Tumbarumba

\[ H + \lambda E = R_n - G_0 \left( -\Delta J_c \right) \]
Summary (1):

• Mass balance of a control volume
• Measurements in surface layer
  ▪ Horizontally homogeneous flow
• Fetch requirements
  ▪ Internal boundary layer
• Instrumentation as a band-pass filter
  ▪ High frequency attenuation – instrument separation, line averaging
  ▪ Low frequency attenuation – averaging period too short
Summary (2):

• WPL corrections for open-path analysers
  ▪ Correct for high frequency loss before WPL

• Closed-path analysers
  ▪ Use mixing ratio relative to dry air
  ▪ Correct for lag & high frequency attenuation
  ▪ Lag for CO$_2$ depends on flow rate
  ▪ Lag for water vapour depends on flow rate & humidity

• Calibrate radiometers

• Change in storage term
  ▪ Measure profiles of $u$, $T$, $q$, $c$ …